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Second Quarterly Progress Report - NASw-871
(Covering Period from May 1 to August 1, 1964)

Project activities and accomplishments are summarized briefly below.

I Participation at Jet Propulsion Laboratories in study of minimum acceptable biological payload. Two project personnel--D. Hitchcock and G. Thomas--spent six weeks of this quarter on site at Jet Propulsion Laboratories as guests of the Biology Section, participating in a study to define a minimum acceptable biological payload for the 69, 71, and 73 opportunities. These Statistical Decision Theory personnel were assigned no explicit responsibilities for portions of the study and have acted primarily as informal consultants to some of the Jet Propulsion Laboratories' personnel. This participation provided an opportunity to obtain more detailed and accurate information about the various experiments being considered than is available in Hartford. During this period, attention was devoted primarily to obtaining information relevant to the two general subproblems identified earlier in the program and described in the First Quarterly Progress Report. These are [data transmission requirements and techniques for comparing the utilities of alternative experiments or alternative payload mixes.] Progress on these two subproblems is summarized below.

1. Data Transmission Requirements - Data communication capabilities of two mission modes were briefly reviewed with Jet Propulsion Lab personnel and brief examinations of the data transmission requirements of four classes of experiments were conducted. The problems of determining optimal data transmission strategies have been found to be more difficult than originally anticipated due to the current status of design of experiments, design and test of prototype experimental apparatus, and specification of mission mode. One class of problems regarding the intervals between observations of a metabolic or growth experiment and sampling errors due to non-random fluctuations of metabolic activity has been studied and the results of this study are in preparation.

The question of bias due to partial or complete equipment malfunction has been explored, and it has been determined that little can be said about optimizing experiments in this respect until prototype design has progressed further. Further theoretical consideration of this problem has been postponed.

2. Techniques for Comparison of Different Experiments - Progress in this subproblem has proceeded along two different lines:

- a. Project personnel have participated in a review of the individual experiments designed to explicate the scientific assumptions implicit in their use as life detection instruments, and to review the availability of standards for interpretation of the data, the possibility of "false positives," and other factors relevant to determining the value of the inferences which may be drawn from data collected by them.

- b. A study of a class of strategies of scientific explorations of Mars expressed in terms of the kinds of properties whose presence can be determined, estimates of the a priori probability that those properties are present on Mars, and estimates of the probability of engineering success of the mission, has been initiated. Given such estimates, alternate property-detecting experiments may be ranked with respect to the probability of detecting the presence of some property; and alternate payload mixes consisting of $n > 1$ property-detecting instruments can also be ranked. Assumptions regarding the dollar and time costs of increasing the probability of engineering success have been made and the effects on the resulting rankings of different postulated tradeoffs are being examined. A specialization of the results to include effects of alternative contamination policies and assumed tradeoffs between the effects of sterilization on reliability of engineering components is planned.

II Simulation

1. The Soup Gedanken program has been completed and debugged. Preliminary runs have demonstrated its utility for exploring effects of variations in the dynamic range and sensitivity of the discriminator which has proved, in this model, to be the most severely limiting feature of the hypothetical experiment. Additional runs are planned for later in the year.

2. String Collection Model. A model of the "sticky string" sample collection system has been designed and is now being programed for repetitive simulation on a 7090.

3. A study has been performed on stochastic models of microbial populations for the purpose of determining whether or not useful a priori conclusions can be drawn from such models to aid in the design of Martian microbe detectors. Very general models of growth can be formulated, but involve a number of unknown probability distributions which are experimentally difficult to measure in terrestrial laboratories. Since no knowledge is available regarding the nature or existence of these distributions on Mars, it was concluded that such generalized models would be of limited usefulness.

For the purpose of computer simulations of growth systems a simplified model was adopted which describes the over-all growth pattern. This model is essentially a discrete analog of the autocatalytic law $\dot{y} = k [y_{\infty} - y]y$.

A series of interim notes are in preparation and will be submitted when complete.

Interim Note 2 - Data Transmission Requirements for Mars Microbe Detector

INTRODUCTION

In the most simple case of a Martian Microbe Detector, we wish to determine whether the change in an optical property such as turbidity follows the law:

$$X(t) = X(0) \exp \{\beta t\} \quad (1)$$

that is, we wish to test the null hypothesis $H_0: \beta = 0$ against the alternative $H_1: \beta > 0$. To this end data $Z(t)$ are collected and an estimate $\hat{\beta}$ at β is computed. If $\hat{\beta} > \omega_c$, where ω_c is some number yet to be determined, we reject H_0 . If $\hat{\beta} \leq \omega_c$, we do not reject $H_0: \beta = 0$ and essentially conclude that no growth has occurred.

In actual fact we shall never observe $X(t)$, but we will observe on earth a quantity:

$$V(t) = G[X(t), \eta_1(t), \eta_2(t), \eta_3(t)] \quad (2)$$

where:

$\eta_1(t)$ = "noise" due to circuit electronics

$\eta_2(t)$ = "noise" due to the digitation of our data to finite words of r bits

$\eta_3(t)$ = biological fluctuations attributable to the failure of eqn.(1) to accurately represent the biological events in the microbial propagator

The problem, therefore, is to extract from the received data $V(t)$ an estimate of β and to construct such other quantities as may be needed to test $H_0: \beta = 0$.

Divide time t into intervals Δt . At the end of the i^{th} interval (provided the circuitry is appropriately designed) we may write:

$$Z_i = \log V_i = Z_{i-1} + \beta \Delta t + e_{i3} \Delta t + e_{i1} + e_{i2} \quad (3)$$

where subscripts 1, 2, 3 indicate electronic, digitization, and biological fluctuations, respectively, and:

$$\begin{aligned} E(e_{i3}) &= 0 & E(e_{i3}^2) &= \sigma_3^2 \\ E(e_{i1}) &= 0 & E(e_{i1}^2) &= \sigma_1^2 \\ E(e_{i2}) &= \Delta^2/2 & V(e_{i2}) &= \Delta^2/12 \end{aligned}$$

$\Delta = C2^{-r}$

C = Full scale voltage for maximum response.

r = Number bits per observation.

E = Expectation operator.

$V(\)$ = Variance of parenthesized quantity.

Equation (3) may be solved recursively in the usual way to yield:

$$Z_N = Z_0 + \beta N \Delta t + \Delta t \sum_{i=1}^N e_{i3} + e_{N1} + e_{N2} \quad (4)$$

An estimate $\hat{\beta}$ of β is given by:

$$\frac{Z_N - Z_0}{N \Delta t} = \hat{\beta} \quad (5)$$

with an expected variance:

$$E(V(\hat{\beta})) = \left[\frac{1}{N} \right] \sigma_3^2 + 2 \sigma_2^2 + \Delta/6 \quad (6)$$

Before discussing any statistical tests it is of interest to examine eqn. (6) from the viewpoint of the optimum assignment of bits per observation. Let T bits be the total number of bits a Mars probe can deliver in its lifetime. Then:

$$Nr = T \quad (7)$$

where N is the number of observations and r is the number of bits per observation. Since $\Delta = C2^{-r}$, we may write:

$$E(V(\hat{\beta})) = 1/N \sigma_3^2 + 2 \sigma_2^2 + \frac{C^2}{6} 2^{-2T/N} \quad (8)$$

which is minimized when

$$N = \frac{0.6T}{\log(0.231C^2T/\sigma_3^2)} \quad (9)$$

As an example of (8) and (9) suppose:

$$\begin{aligned} C &= 1400 \text{ nano amp (na)} \\ \sigma_2^2 &= 0.5 \text{ (n.a.)}^2 \\ \sigma_3^2 &= 100 \text{ (n.a.)}^2 \\ T &= 1000 \text{ bits} \end{aligned}$$

Then $N \sim 90$, $r = 11$ and $\Delta = 2^{-11} = 0.7$ is the optimum allocation of bits. More usefully, let σ_3^2 , a ground based large sample estimate obtained in terrestrial studies, be available. Let σ_3^2 be expressed as a binary divisor of $0.231C^2T$, i.e.

$$\sigma_3^2 = 2^{-u} (0.231) C^2T \quad (10)$$

Then the optimum assignment is:

$$\begin{aligned} r &= u/2 \\ N &= 2T/u \end{aligned} \quad (11)$$

The preceeding treatment ignores the fact that the statistical test of H_0 is of the form:

$$\hat{\beta}^2 / S_{\hat{\beta}}^2 > F_{\alpha} \quad (12)$$

where $S_{\hat{\beta}}^2$ is a sample estimate of $V(\hat{\beta})$ and F_{α} is the critical variance ratio at the α level of significance.

To form an estimate $S_{\hat{\beta}}^2$ form a table of first divided difference D_i in Z_i so that:

$$D_i = \frac{Z_i - Z_{i-1}}{\Delta t} \quad (13)$$

Compute:

$$S_{\hat{\beta}}^2 = \frac{1}{N(N-1)} \sum_{i=1}^N (D_i - \bar{D})^2 \quad (14)$$

where \bar{D} denotes the sample mean:

$$\bar{D} = \frac{1}{N} \sum_{i=1}^N D_i$$

The expected value of S_p^2 is:

$$E(S_p^2) = \left[\frac{1}{N} \right] \sigma_3^2 + \frac{2\sigma_2^2}{(\Delta t)^2} + \frac{\Delta^2}{6(\Delta t)^2} \quad (15)$$

which, unlike eqn. (6), contains the parameter Δt according to eqn. (15) values of $(\Delta t)^2$ small relative to σ_2^2 or Δ^2 will inflate S_p^2 .

If G denotes the duration of an experiment it can be shown that an absolute minimum to (15) can be obtained by dispatching $N/2$ experiments each of which makes two observations, one at $t=0$ and one at $t=G$ and utilizes T/N bits for each observation.

If only one experiment can be dispatched, then the optimum number of observations is a root of:

$$-\frac{1}{N} \sigma_3^2 + \frac{4N\sigma_2^2}{G^2} + \frac{C^2}{6G} \left\{ 2N2^{-2T/N} + 2KT2^{-2T/N} \right\} = 0 \quad (16)$$

An approximate optimum can, however, be obtained as follows:

If σ_2^2 and Δ^2 are much smaller than σ_3^2 , the variance will be dominated by σ_3^2/N . A value of N can be estimated from terrestrial trials to yield an experiment of suitable sensitivity. From an estimation of G , an estimate of Δt can be formed such that σ_2^2 and Δ^2 are small relative to it. From the value of Δ the word length r can be guessed. This value, along with T and N , can be used to solve (9) to get an estimate of C .

Discussion

In a recent note entitled "Statistical Problems Concerning the Mars Microbe Detector," J. Weiss (Ball Brothers Research) has discussed the problem of optimum bit requirements using a model different from (3). Weiss's model makes no allowance for the influence of biological fluctuations e_3 , nor for the fact that such fluctuations will propagate themselves throughout the whole course of the experiment. Although it is feasible to examine the optimization problem in greater detail than was done above, it seems more fruitful to examine the influence of expected sampling variation arising from replicated experiments rather than one experiment and the assignment of bits under growth laws more general than that of eqn. (1). This will be done in later notes.


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